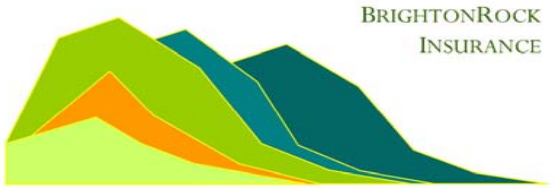
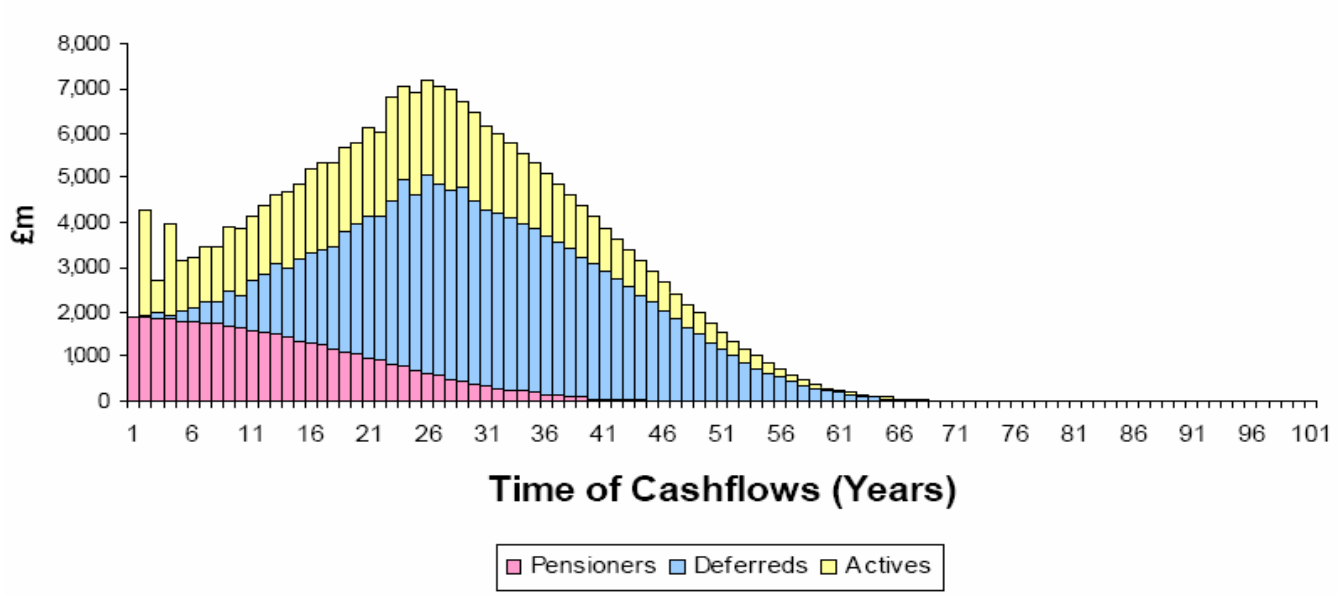


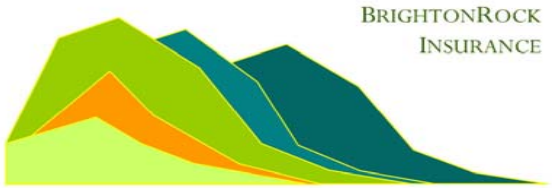
Risk Measures in Finance



An Introduction

Con Keating

Amsterdam 2008



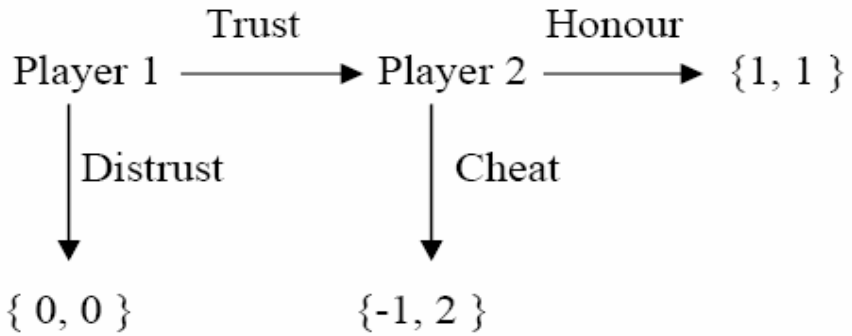
Uncertainty

- The problem of the *double exchange of wants*
- Bacon producer must find potato farmer
- This is uncertainty
- *Money* resolves this
- Finance and Money are inextricably linked
- This implies at best a very limited role for deterministic analysis
- But not all uncertainty is risk
- Risk is a subset of uncertainty
- But not precisely known – it has unknowable boundaries
- We are incapable of knowing all possible future states and of being able to assign probabilities to each state
- Frank Knight – wider indeterminacy of the truth of any broad proposition
- If we have demonstrably correct ways of assigning all probabilities, profit can only be derived from scarcity

BRIGHTONROCK INSURANCE

Trust

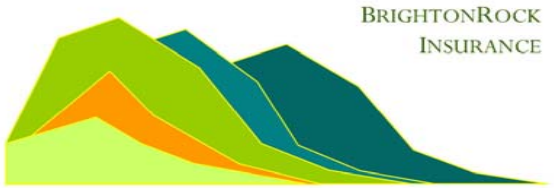
Kreps Trust Game



The static game equilibrium is {0,0} – Player 2 cheats, and Player 1 distrusts.

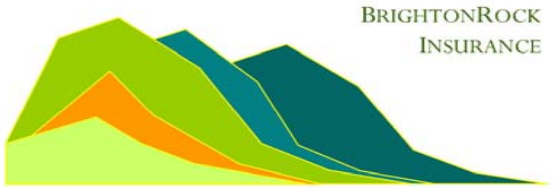
But when repeated, the dynamic equilibrium, trust and honour {1,1}, emerges.

Note the results in static analysis differ from those arising from the dynamic



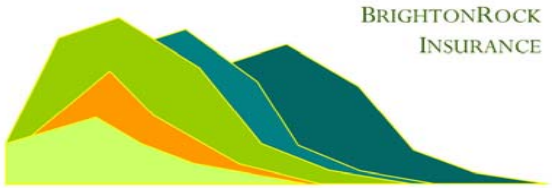
Boundaries to Uncertainty

- The boundedness of uncertainty is essential to the possibility of decision, but the boundaries are uncertain
- A (concealed) pretence is necessary, but
- Behavioural finance tells us that we consistently over and underestimate extreme events
- The problem is well known in probability (sampling) theory
- The sample (our experience) is drawn from within the boundaries of the true distribution
- Extremes are unreliable



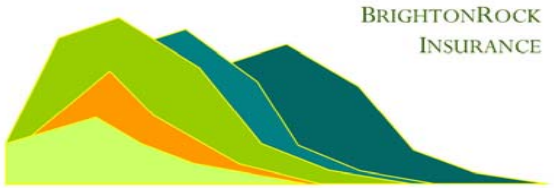
Two central models

- Arrow-Debreu – state prices and Pareto optimal equilibria
- But there is no description of how these come to be formed
- And the difficulty is that outside of equilibrium, there must be some agent not optimising, but optimisation is a necessary assumption for all.
- CAPM and BSM – Correct pricing may be logically derived from correct evaluation but that requires a demonstrably correct way of identifying all relevant possible events and assigning probabilities.
- There can be no surprises - even new knowledge
- Money is always perfectly available – actually it doesn't exist.
- There is an all powerful arbitrageur, with adequate funds
- And the ability and opportunity to identify and exploit every opportunity
- And then there is no trade in this market – Grossman and Stiglitz



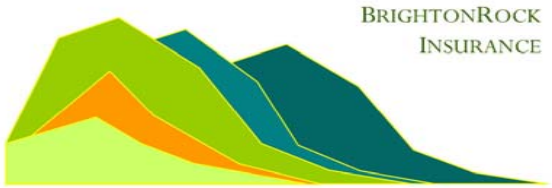
Ockham's razor

- We need models to organise our thought and knowledge
- It is necessary to exclude some things
- Like Ockham, we prefer parsimony.
- But Friedman: "*A hypothesis is important if it 'explains' much by little……if it abstracts the common and crucial elements from the mass of complex and detailed circumstances……and permits valid predictions on the basis of them alone. To be important, therefore, a hypothesis must be descriptively false in its assumptions.*"
- To which Samuelson countered:"[It is]…*fundamentally wrong that unrealism in the sense of factual inaccuracy even to a tolerable degree of approximation is anything but a demerit for a theory or hypothesis. …The fact that nothing is perfectly accurate should not be an excuse to relax our standards of scrutiny of the empirical validity that the propositions of economics do or do not possess.*"



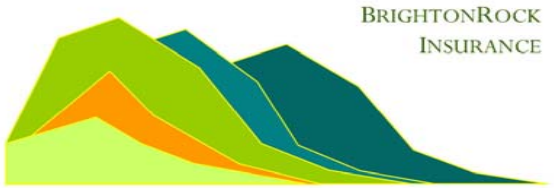
Models

- There are questions with all financial theories as to whether they function as cameras, recording empirical regularities or as engines influencing performance.
- Most important for these latter theories are the prescriptive actions they generate to reinforce themselves. Or as the German sociologist Max Weber expressed it: "*To seal the ideological bondage"*
- Better is to use models to parse data into information and noise
- No assumption of a true model is needed
- The objective becomes to extract the useful and learnable information with a model class suggested
- By defining learnable information, we can also compare any two models



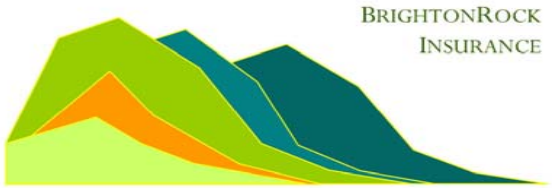
Information Parsing

- Consider the following data
- An alternating binary string 010101…
- If our model is Bernoulli with the probability of each symbol (0,1) one half
- Then this string is all noise, and the information is zero
- But if our model is first-order Markov, with the conditional probability of symbol (1) unity in State 0, and of symbol (0) unity in State 1,
- Then the entire string is information and there is zero noise
- So from the same data different model classes can extract entirely different properties from the same data, while imposing different constraints
- And this is the very purpose of modelling.



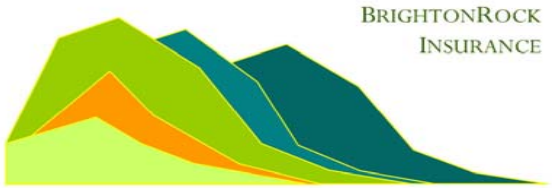
Information Theoretic Modelling

- Can be directly related to probability and offer some useful further insights
- The extreme events which are so problematic in practical risk management are precisely the events which are information rich
- Further, in general terms, maximisation of the probability of the data with respect to the class of model involves a process which penalises the number of parameters
- In the spirit of Ockham
- And leads to optimally “distinguishable” models
- Which have application in hypothesis testing and confidence estimation
- In particular the Bayes Information Criterion (BIC) emerges naturally but rather more fully than the only asymptotically effective version we are familiar with.



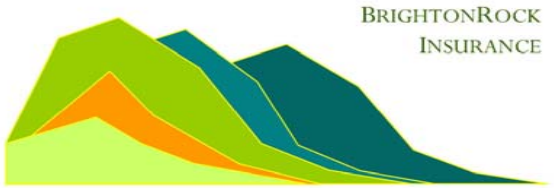
Information

- With perfect information institutions don't matter, nor does history or the distribution of wealth.
- Imperfect knowledge of price, quality and effort has no effect
- Given the production technology, initial endowment and preferences all future is determined.
- The Arrow-Debreu world is one of perfect information – beliefs cannot be endogenous and cannot change
- And that precludes investing in information discovery
- Imperfect information has major consequences, even if small.
- It limits markedly the domain of the law of supply and demand, the law of diminishing returns, the law of the single price and the efficient markets hypothesis.
- In general though it never pays to invest in just a little information discovery.



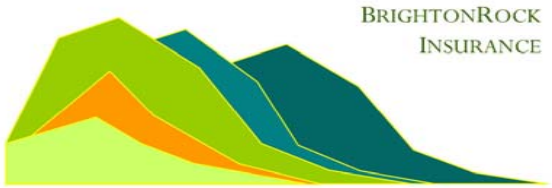
Mixed games

- A distinct co-ordination feature of financial markets – Keynes' newspaper competition in which "*the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick not those faces which he himself finds prettiest, but those he thinks likeliest to catch the fancy of other competitors, all of whom are looking at the problem from the same point of view.*"
- Francis Galton's study of the wagers placed on the weight of an ox at a fair in 1906 is relevant in the context of aggregation of individual beliefs – the average guess was 1,198 pounds and the actual weight was 1,197 pounds.
- However these wagers had one important property; they were independent – no-one knew anyone else's wagers. But when a crowd is permitted to know the wagers of others, we admit the possibility of strategic behaviour.



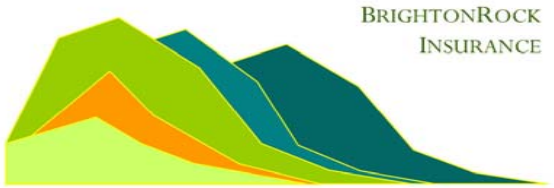
Mixed games

- Strategic behaviour and Keynesian co-ordination have the effect of making endogenous to the market new sources of risk and uncertainty; this lowers the role of true information in price formation.
- In Galton's example this might consist of placing wagers close to those of the experts, the slaughter-men and butchers, or equally of relying on rating opinions in a more recent context.
- The possibilities for hysteresis or feed-back in such situations are obvious – and can result in very strange equilibria indeed. Certainly there is no longer any reason to believe that the law of large numbers will apply to this market, or that its elementary statistics will be relevant, adequate descriptions.
- It is evident that financial markets are mixed games – they are partly games against nature and partly games against others. Uncertainty and risk have both exogenous and endogenous sources.
- In such situations the all powerful arbitrageur is as likely to behave in a destabilising predatory manner as to arbitrage prices back to fair value.



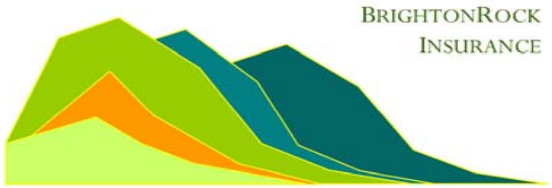
Endogenous risk models

- Morris and Shin, Danielsson and Zigrand
- Generalised Games
- More advanced than today's introductory exposition



Accounting standards
Assets and capital assets

- Today's market price may, perhaps, reflect current information perfectly, but, even with the addition of rational expectations, that does not mean that it embodies perfect information with respect to future prices.
- That would require perfect foresight.
- A consumption good may be consumed immediately; the information content of its current market price is close to complete.
- Though the option to defer consumption needs a little thought, as do alternate possible uses as a store of value.
- The value relevance of a market price is high for such assets
- By contrast, the information content of a market price for a capital asset is at best limited to that known today, which is far from complete over the life of that capital asset

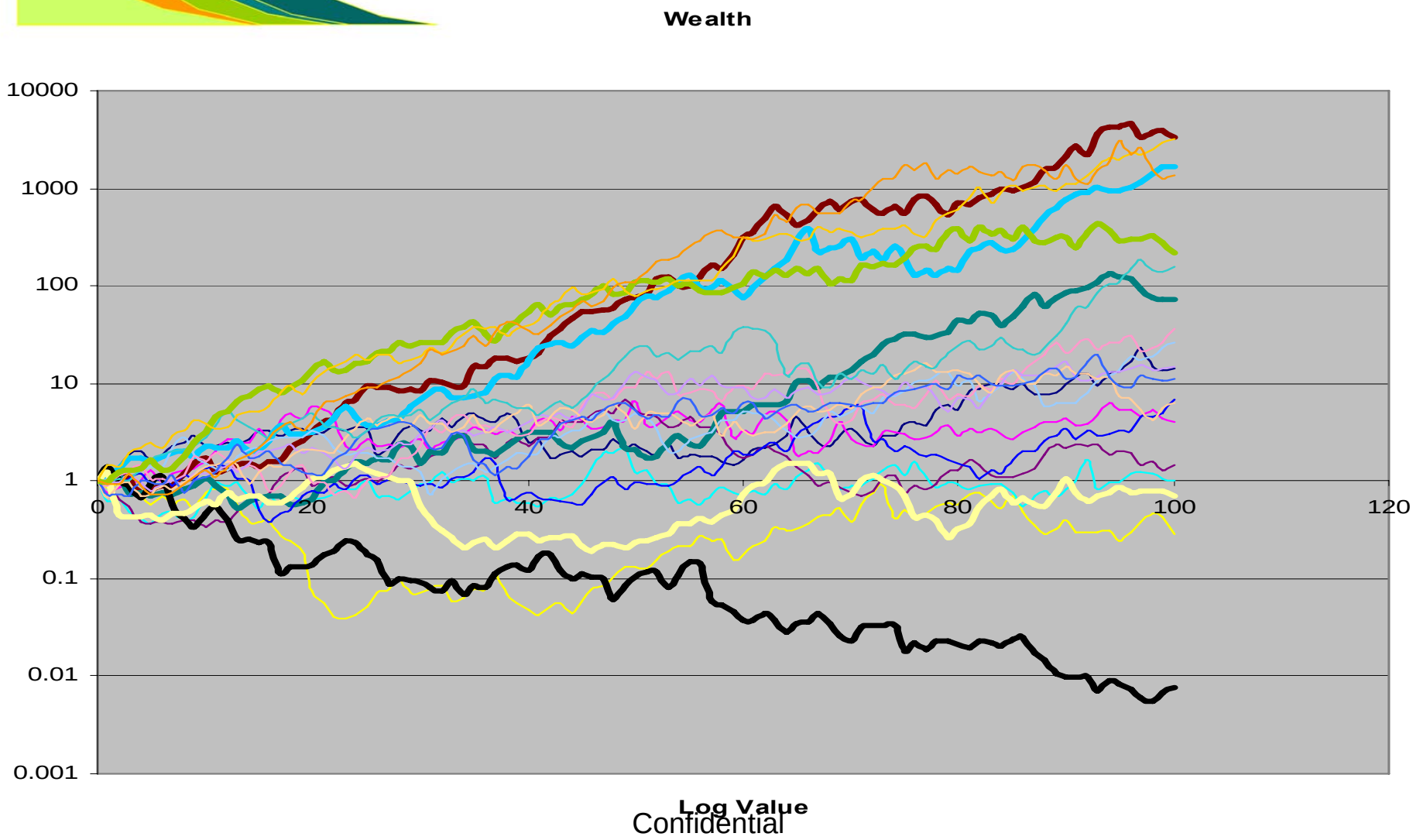


Capital assets

- Of course, for very long lived capital assets the information content of today's price may actually be very small and the value relevance low.
- Obviously as the time dimension increases, so too does the potential information incompleteness of a market price today with respect to the future value.
- Each and every price may have differing information content, though they all share a common set, today's available information.
- The value relevance of today's price may also differ for each and every owner of the capital asset.
- Increasing the size of today's information set, that which is available today, will not remedy this incompleteness as that is based predominantly in future information.
- The more incomplete the information content of a price the greater is the potential for strategic behaviour, and far more substantial and sustained departures of the market price from true value.

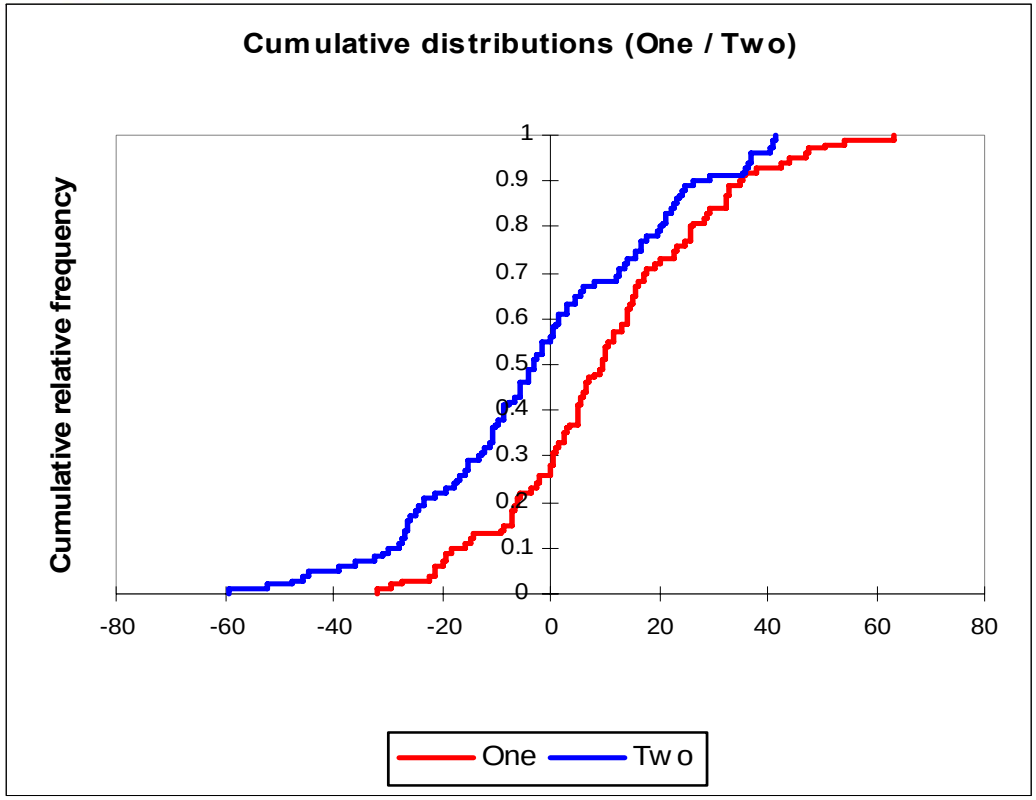
BRIGHTONROCK INSURANCE

Fooled by Randomness

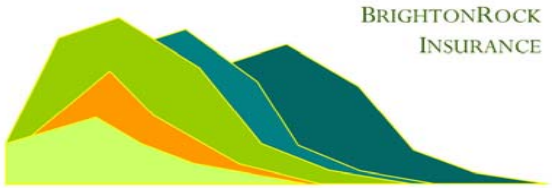


BRIGHTONROCK INSURANCE

Tests



KS Tests –less than 2% likelihood they are draws from the same



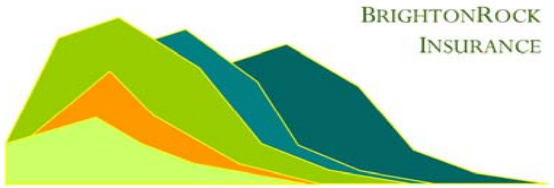
Tests (9 & 19)

Difference Means	**-0.288**
z (Observed value)	**-0.112**
z (Critical value)	**1.960**
p-value (Two-tailed)	**0.911**
alpha	**0.05**

Ratio (Fisher)	**1.215**
F (Observed value)	**1.215**
F (Critical value)	**1.486**
DF1	**99**
DF2	**99**
p-value (Two-tailed)	**0.335**
alpha	**0.05**

Difference Means	**-0.288**
t (Observed value)	**-0.112**
t (Critical value)	**1.972**
DF	**198**
p-value (Two-tailed)	**0.911**
alpha	**0.05**

Levene's test (Mean) / Two-tailed test:	
F (Observed value)	**0.932**
F (Critical value)	**0.001**
DF1	**1**
DF2	**198**
p-value (one-tailed)	**0.336**
alpha	**0.05**



Non-parametric Tests (9,19)

Mann-Whitney test	
U	5089.000
Expected value	5000.000
Variance (U)	167500.000
p-value (Two-tailed)	0.829
alpha	0.05

Wilcoxon signed-rank test	
V	2591
Expected value	2525.000
Variance (V)	84587.500
p-value (Two-tailed)	0.823
alpha	0.05

Test interpretation:

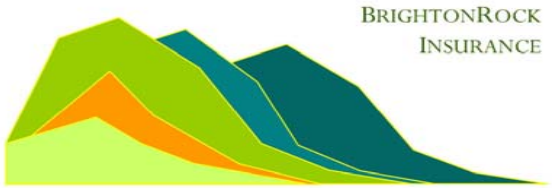
H0: The location difference between the samples is not significantly different from 0.

Ha: The location difference between the samples is significantly different from 0.

As the computed p-value is greater than the significance level alpha=0.05, one should accept the null hypothesis H0.

The risk to reject the null hypothesis H0 while it is true is 82.88%.

N+	51
Expected value	50.000
Variance (N+)	25.000
p-value (Two-tailed)	0.920
alpha	0.05

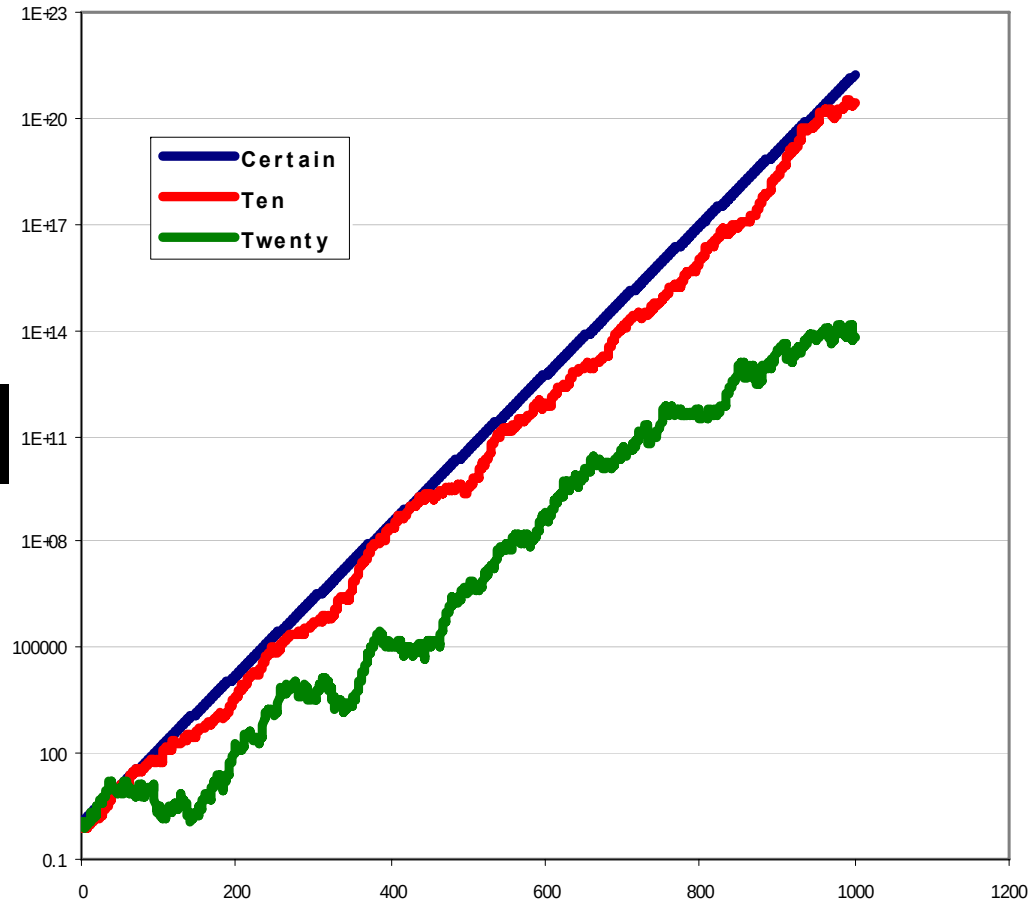


The message

- The power of statistical analysis in a financial context is low
- Most of the claims of skill are unverifiable
- Falsification is most often by disaster
- But perhaps we prefer the lucky to the unlucky
- Let’s return to risk and return.

BRIGHTONROCK INSURANCE

Certainty Equivalent Premium



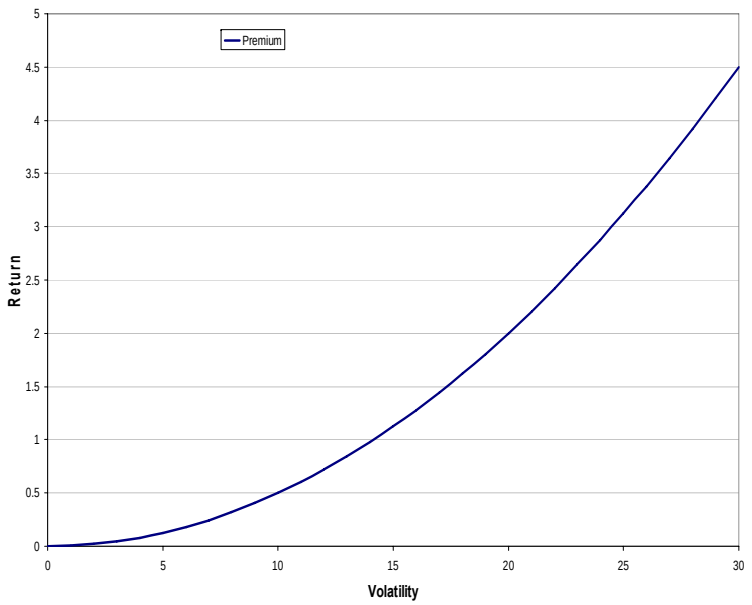
Geometric Mean = Arithmetic Mean – 0.5*Variance

10% volatility = 0.5%
20% volatility = 2.0%

Property of repeated games

Volatility / Risk has a cost

Risk



Demanding the certainty equivalent "premium" is not risk averse behaviour

How does the certainty premium behave?

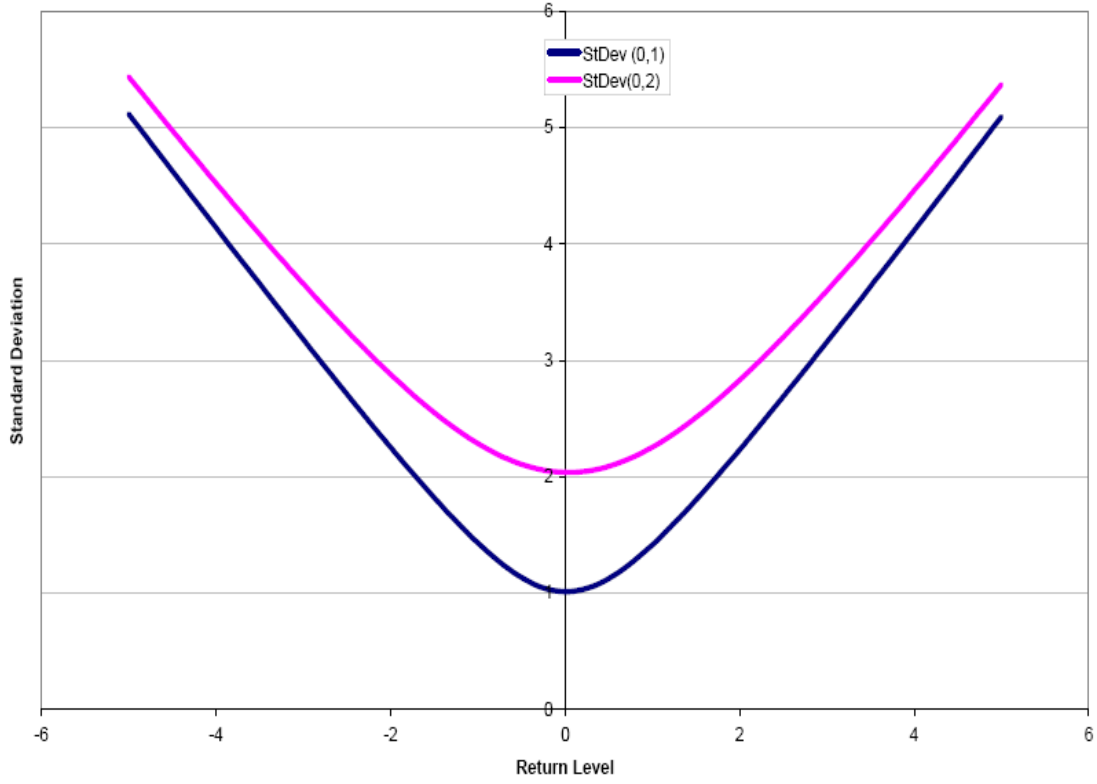
It is non-linear – quadratic

when returns are normal.

But this creates a **real** problem for the linear models of market risk – such as the CAPM

Local and Global Volatility

BRIGHTONROCK INSURANCE



Global Measure: volatility about the mean

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

Local Measure: volatility about a point

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - r)^2}$$

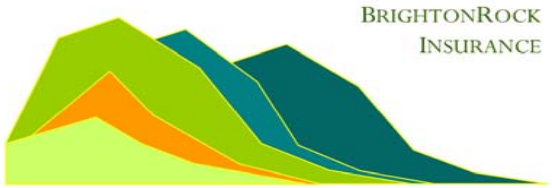
BRIGHTONROCK INSURANCE

Why does this matter?

- Erb & Harvey – Geometric Returns
- Commodity index returns
- Diversification matters in more than a risk mitigation sense.
- It enhances the geometric return of the portfolio
- And this result is not dependent upon risk premia
- In the case of jointly normal assets
- Expected equally weighted rebalanced portfolio diversification return is:

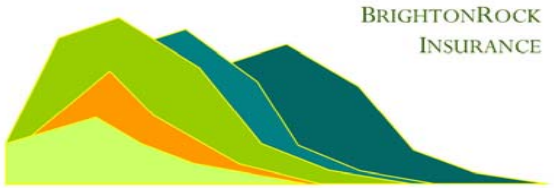
$$E\{\text{EWRPDR}\} = \frac{1}{2}\left(1 - \frac{1}{K}\right)\bar{\sigma}^2\left(1 - \bar{\rho}\right)$$

Strategies now matter



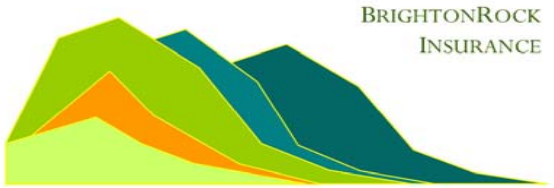
A choice game

- Choose between receiving
- A) €100 with certainty
- B) €200 / €0 on the flip of a coin
- And now paying
- C) €100 with certainty
- D) €200 / €0 on the flip of a coin
- Most choose A and D
- Reversal of attitude
- Does your choice change with repetition?



Risk and Value

- The game illustrates the differing role of uncertainty for assets and liabilities.
- For our asset uncertainty reduces current and future value.
- This does not require “risk aversion”
- It is a property of repeated games
- But that which hurts our asset helps our liability
- Liabilities – the long term aspect is good not bad
- Equity is preferable to short term bank loans
- A consequence – the more certain a liability is, the greater its current cost
- A liability differs from its asset counterpart in location and symmetry
- Multiplication by -1, a rigid rotation
- Not a translation on the returns line



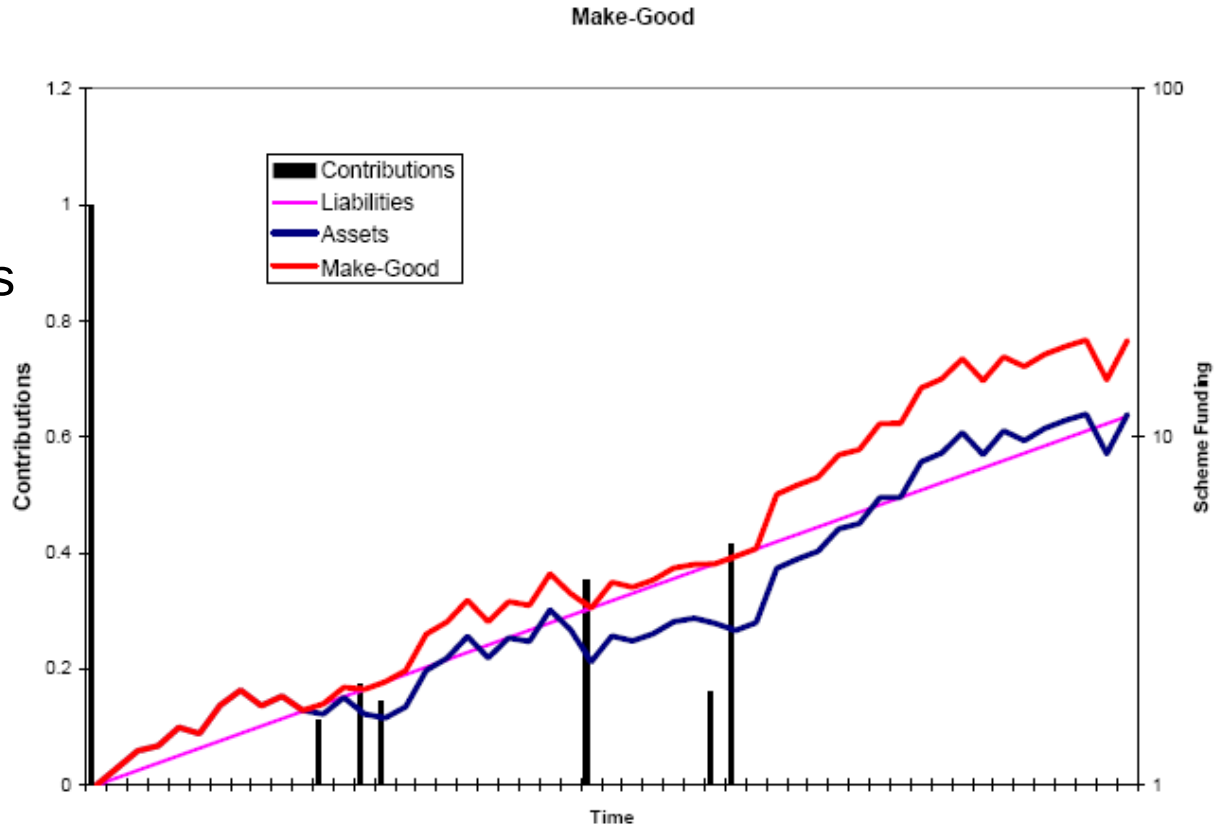
Some further volatility effects

- Excess volatility in financial markets is one of the great puzzles of financial theory
- If we expressly introduce this into scheme funding requirements, for example by the use of target funding levels such as the OECD's recommended 100% we see that the costs of the scheme rise
- This is an explicit cost of myopia, short-term-ism.
- We show a fifty year scheme with liabilities increasing at 5% per annum deterministically and assets increasing at 6% pa with volatility at 10%
- We also show the *"make-good"* portfolio which requires additional contributions whenever the scheme falls below 100% funded.
- Perhaps the most important volatility effect is perversely the advent of Liability Driven Investment

Funding Rules

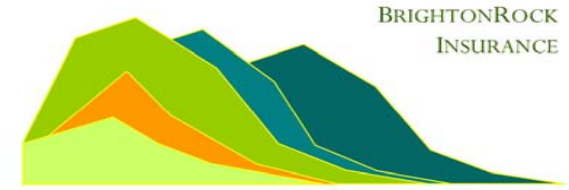
The additional contributions needed to maintain full funding have a net present value of 46% of the fund's initial value

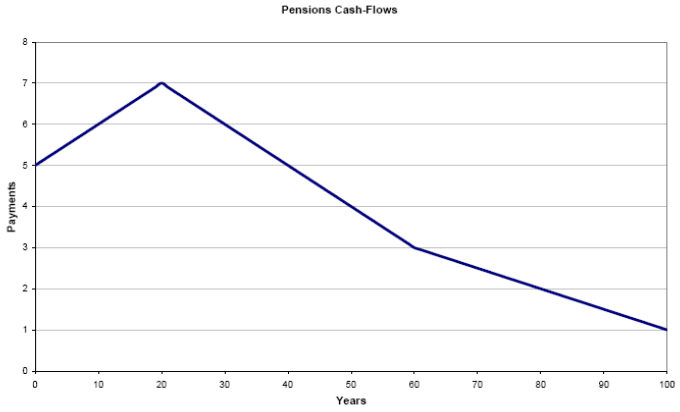
And this is raised in times when the cost of capital is higher than the average



We can also show that deficit based rules are, in general, inefficient

Survival Times



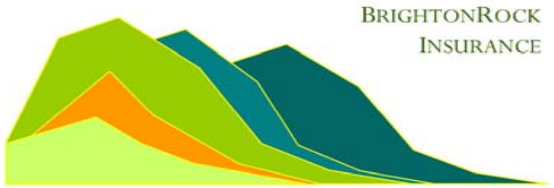


Cash-flow Life (Years)				
	Interest rate			
	Three	Five	Seven	Ten
50% Funded	18	13	10	7
60% Funded	22	17	13	10
70% Funded	29	21	17	13
80% Funded	36	27	22	17
90% Funded	49	38	30	23
100% Funded	NM	NM	NM	NM

The interesting feature of this stylised model is that the lower interest rates are, the longer one has to rectify deficits

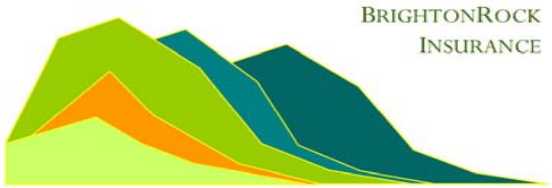
It also suggests that scheme specific funding requirements for pensions should be interest rate dependent.

Finally it demonstrates the inadequacy of the view of risk management as concerned only with the immediate



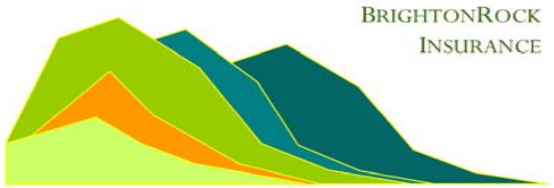
Expectations

- There is much confusion over risk measures
- In the CAPM risk is the average variation of return from the expectation, which is itself the average return.
- These are respectively the second and first moments of the distribution
- The first “moment” is also known as the location, the mean
- And such simple expectations are simply additive
- $E(X+Y) = E(X) + E(Y)$
- Portfolio diversification effects are absent
- But for the second moment
- $E(X+Y)^2 \leq E(X)^2 + E(Y)^2$
- Volatility is fundamentally sub-additive
- But…



Confusion

- We often refer, wrongly, to first moments as risk
- Examples are: Longevity Risk & Credit Risk
- In the case of longevity risk we are talking of error in (first) expectation
- The model is mis-specified and does not allow adequately for increasing lifetimes
- There is no benefit to risk pooling
- Credit Risk is also a first moment
- It should not be expected to diversify away



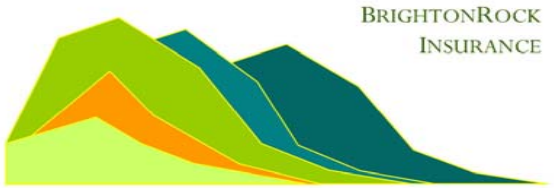
Diversification of Corporates

Corporate spreads are largely a compensation for bearing credit risk, and one reason why they are so wide is that losses from default can easily differ substantially from expected losses.

Moreover, such risk of unexpected loss is evidently difficult to diversify away.

"As corporate bond portfolios go, one with 1,000 names is unusually large, and yet our example shows it could still be poorly diversified in that unexpected losses remain significant."

Reaching for yield: Selected issues for reserve managers
Remolona and Schrijvers, BIS Quarterly Review, Sep 2003



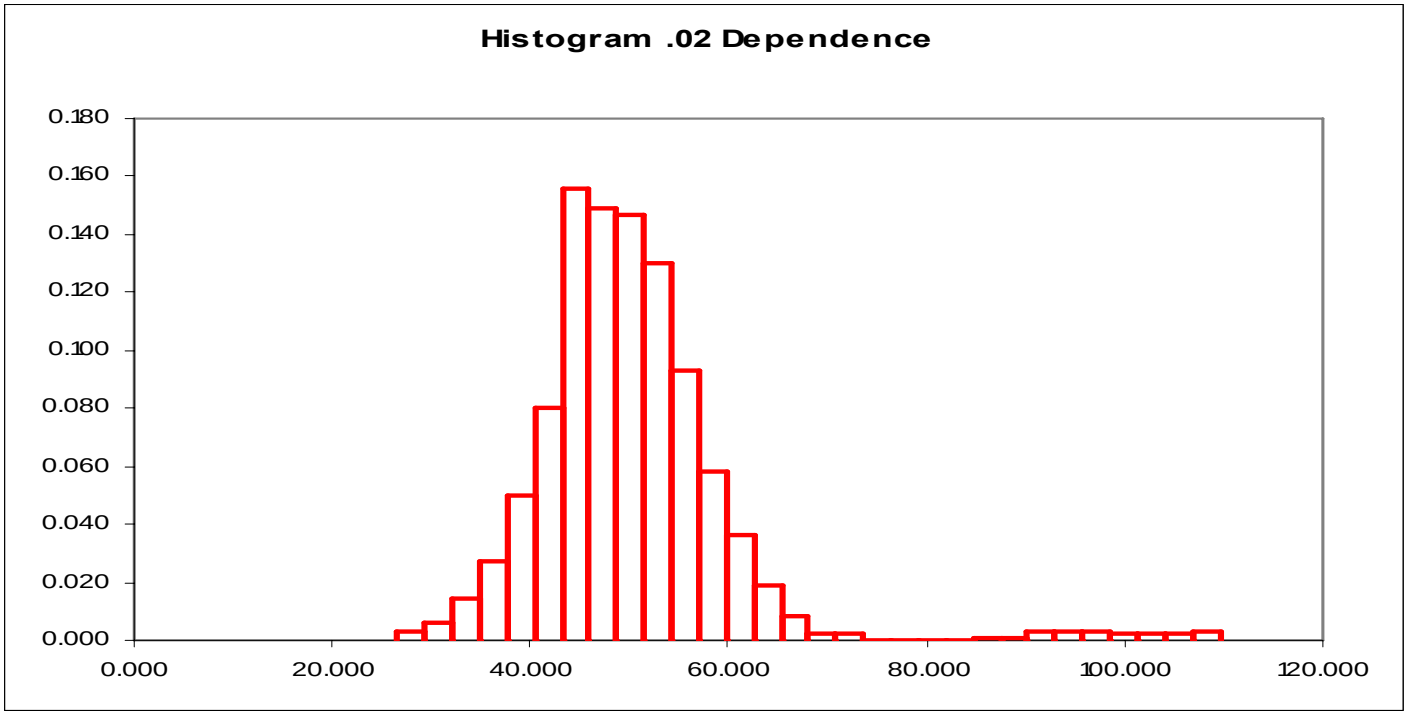
Dependence structure

- Correlation is a linear measure of dependence in data
- Relevant when our measure of risk is volatility
- But the presence endogenous risk may invalidate this simplicity
- Copulas can be used to deliver more complex fixed structure in dependence
- But their selection can be problematic
- More general is common factor dependence
- This may in specific circumstances be negative
- Consider the situation where the failure of one firm leaves the remaining more profitable
- More common is mutual dependence
- And this can have effects on the first moment

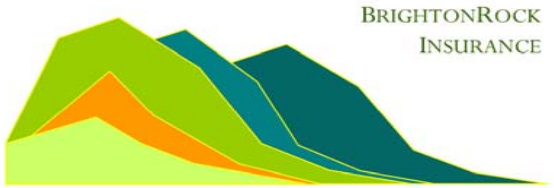
BRIGHTONROCK INSURANCE

Even small correlation can be harmful to your health

A distribution of defaults with .02 correlation



98% independent 2% dependent



Correlation and Dependence

Higher moments are needed to capture dependence.

Correlation tells one little about the shape of the joint distribution

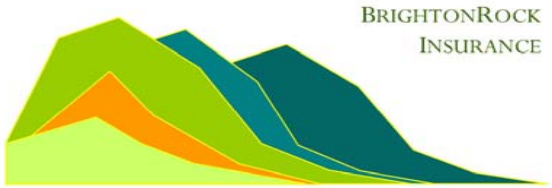
Copulae are little better.

The presence of common factors tells much about dependence.

Common Factors diversify slowly if at all

The limits to (additive) diversification are well known

But in the presence of common factors diversification may be slow and inefficient.



Common Factors

In the presence of common factors, tails can be arbitrarily thick.

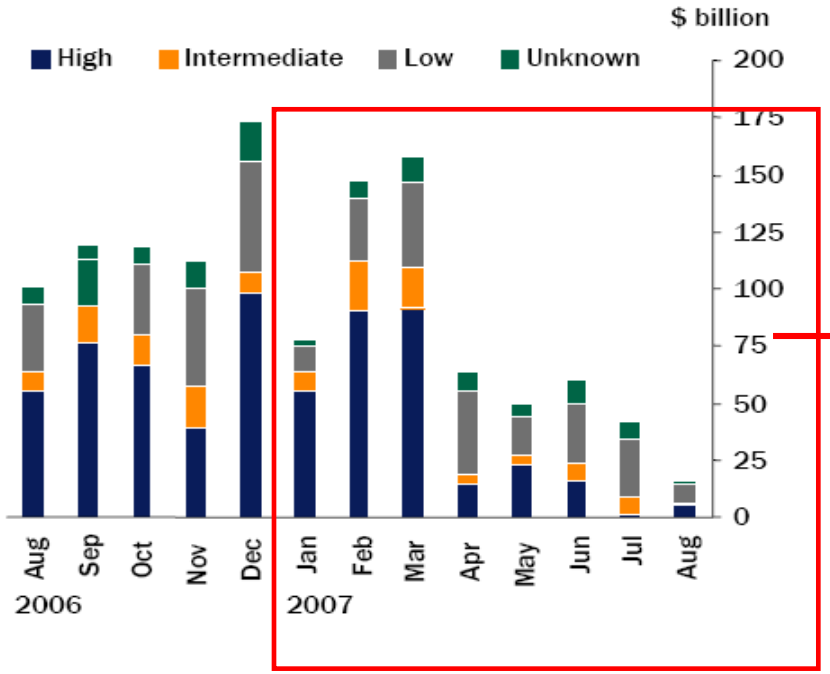
In the previous illustration, 100 defaults occur 5 standard deviations from the mean.

This is the free lunch associated with CBO transactions

Diversification score construction cards are flawed in this regard.

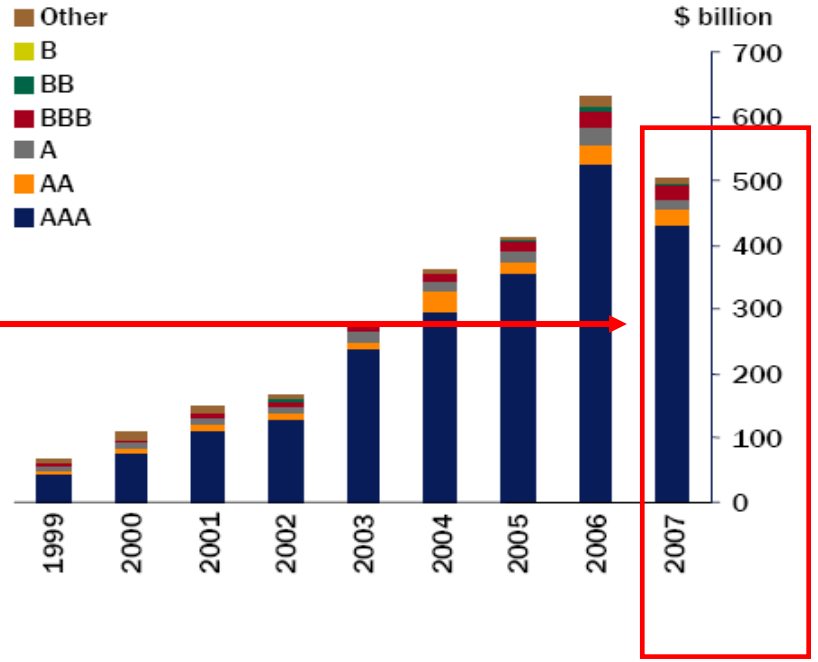
BRIGHTONROCK INSURANCE

Credit Quality Of Underlying Collateral In CDOs/CLOs

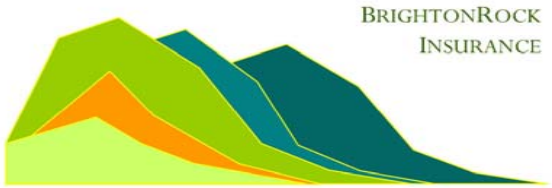


Sources: J P Morgan and Bank of England calculations

Credit Quality Of Structured Finance Investments



Sources: Lehman Brothers and Bank of England calculations



Factor modelling

- In simplest form just matrix manipulation
- Beware of inferential cause and effect problems
- Non-linear models are possible
- The key, as with all econometrics, is careful examination of the residual (error) distribution
- Causal chain analysis is preferable
- Advanced pattern recognition techniques may be appropriate
- In modern application, under-determined linear systems may be resolved with high confidence
- A determined system is one in which we have as many equations as unknowns
- An underdetermined system where the matrix is sparse may be resolved for practical financial application
- Multi-collinearity of data is frequently encountered in finance

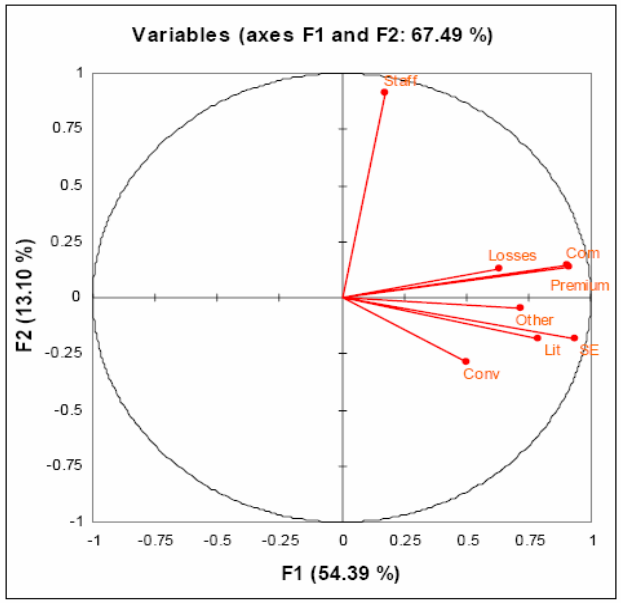
BRIGHTONROCK INSURANCE

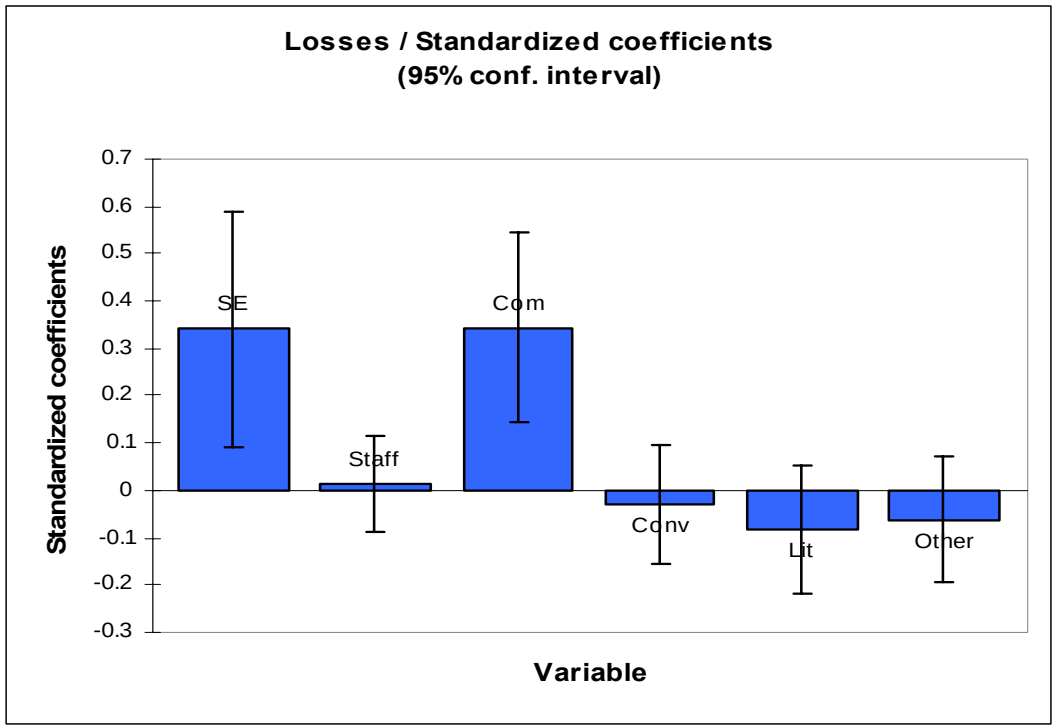
Factor Modelling
A caution

Which can result in nonsense

The data exhibit multi-collinearity

Standard OLS multiple linear regression



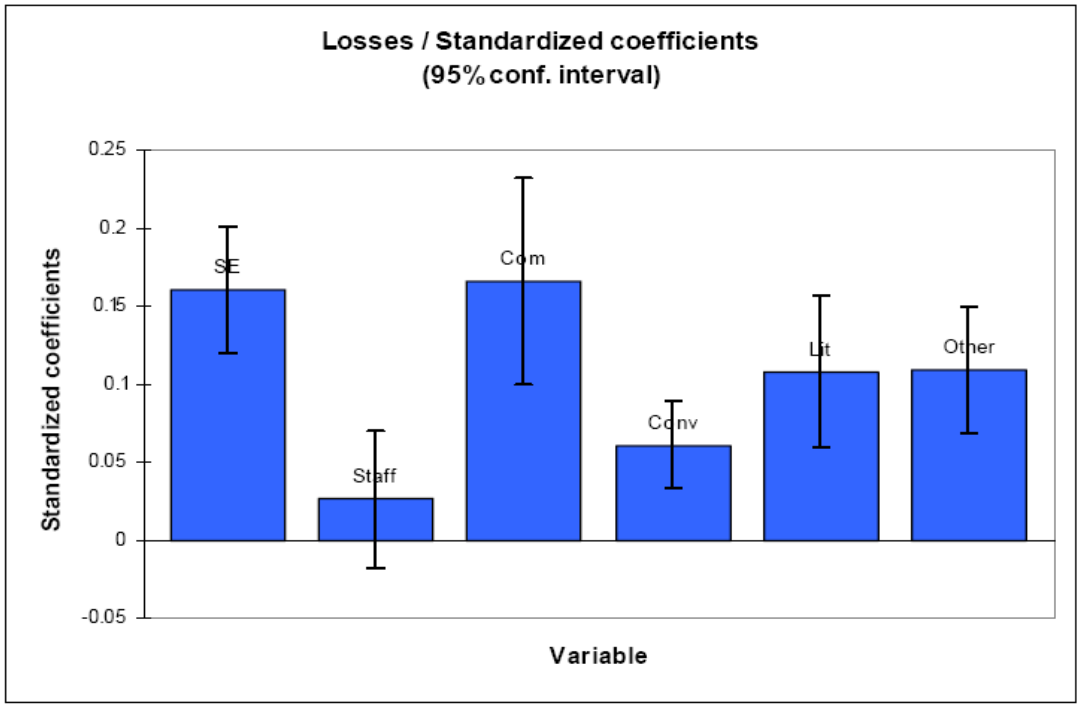


BRIGHTONROCK INSURANCE

Losses: a simple model

A linear model estimated by robust methods

$$\text{Claims} = 16900 + 1138\,\text{SE} + 667\,\text{Staff} + 0.003\,\text{Comm} + 0.005\,\text{Conv} + 0.004\,\text{Lit} + 0.008\,\text{Other}$$



The explanatory power is low: R-Squared 26%

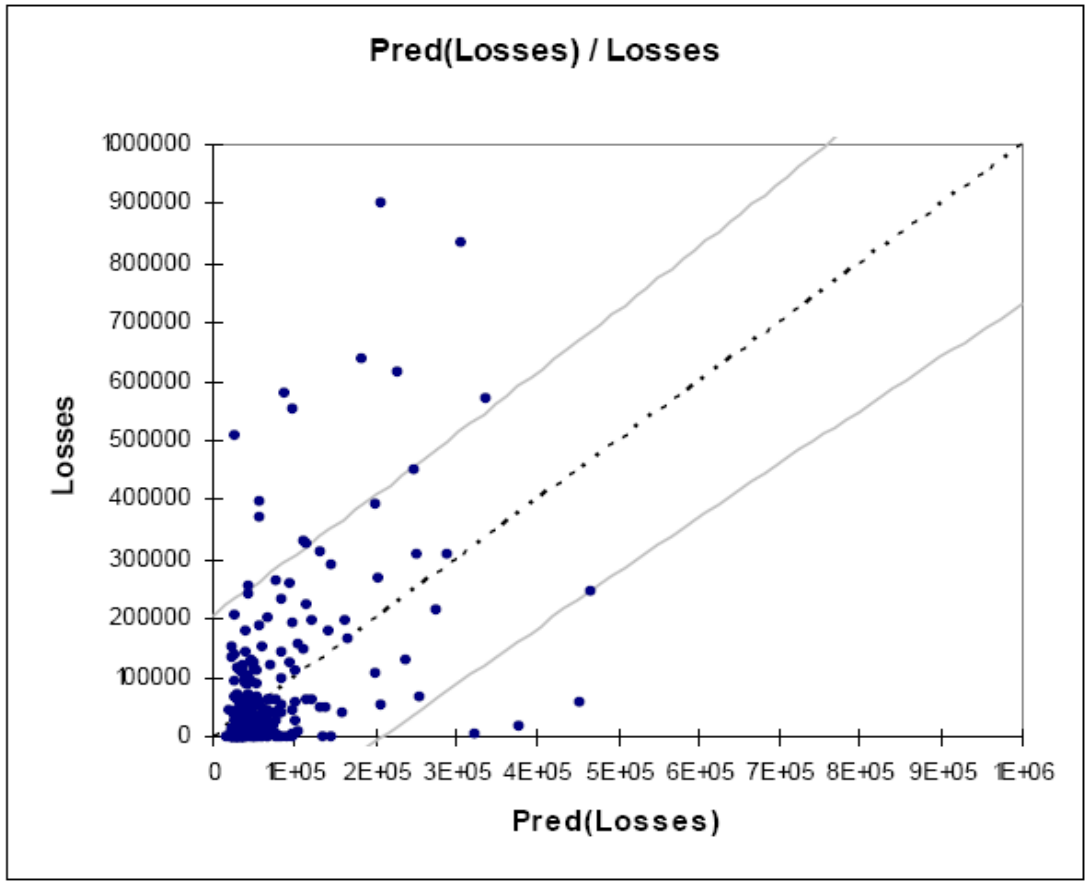
Non-linear models do not improve this materially

Non-Parametric Models do not improve this materially

A Hypothesis: Most losses arise from accidents not systematic cause

BrightonRock Insurance

The claim model is far from perfect

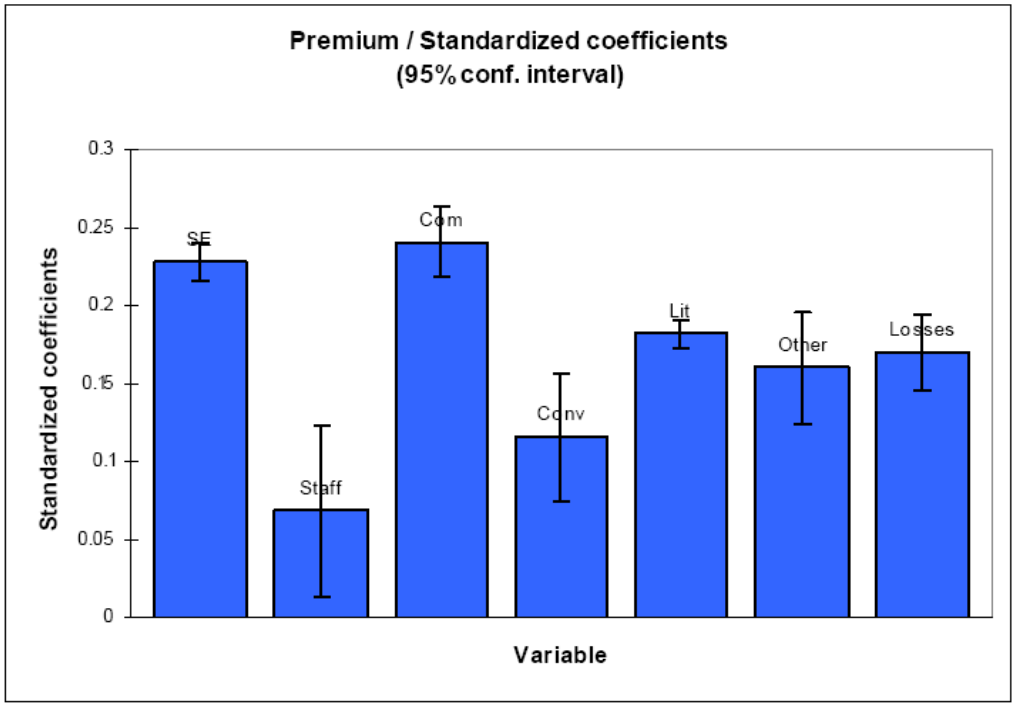


But of the 13 exceptional loss firms, 12 are class 5 risks

BRIGHTONROCK INSURANCE

Traditional Premium Market Model

A simple linear model again



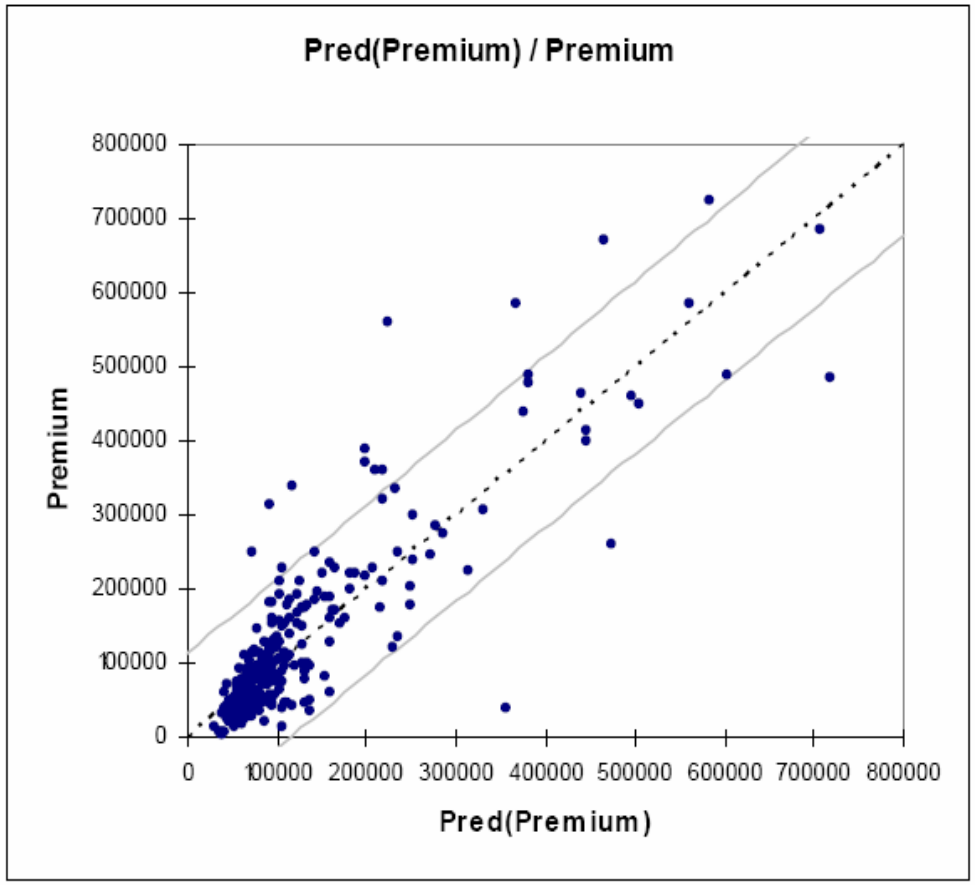
Explanatory power is good: 76.2%

Model minimum premium target is £30,597

The market prices using risk factors more heavily than the factors themselves justify

BrightonRock Insurance

Market Premium Model



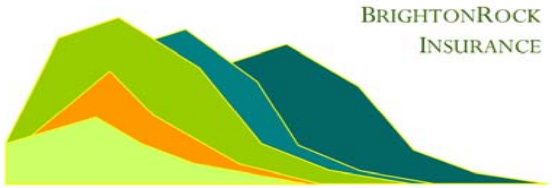
The market extracts higher premiums from 11

We expect 17
Ex ante

The market charges lower premiums to just 3

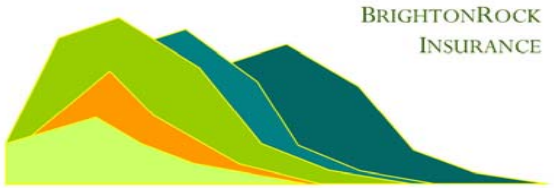
Again we expect 17
Ex ante

Solicitor negotiation power is weak



Uncertainty and Risk

- The world is an uncertain place and this arises from the arrow of time
- We observe only one outcome
- Many were possible
- Probability
- Kolmogorov's axiomatic framework
- A property of semi-groups
- Naturally occurring in systems far from equilibrium – both entropy creation and self-organisation can occur.
- For us relative frequency of events suffices – but we observe only one outcome
- Stochastic versus Deterministic
- The role of time means that risk and dynamics are integrally related.



Part 2

- Will build from here
- And consider many of the standard risk measures in use
- And many of their limitations

- Questions
- Con.Keating@BrightonRockGroup.co.uk

BRIGHTONROCK INSURANCE

If time permits – Structured Finance

The price of an asset with pay-off X is given by

$$P = E[MX],$$

where M is the stochastic discount factor.

And so $P = E[M]E[X] + Cov[M,X]$.

With power utility, this M would be

$$M = \left[\frac{C_{t+1}}{C_t}\right]^{-\gamma}$$

Where C is consumption.

Now $E(M) = \frac{1}{1+r}$, where r is the risk free rate, so

$$P = \frac{E[X]}{1+r} + Cov[M,X]$$

BRIGHTONROCK INSURANCE

And the corporate bond application

Now let $X = 1 - Loss$, so

$$P = \frac{E[1-Loss]}{1+r} + Cov[M, 1-Loss] = \frac{1-E[Loss]}{1+r} - Cov[M, Loss]$$

Now consider two bonds (1,2), both with the same expected loss,

$$E[Loss(1)] = E[Loss(2)]$$

And (1) is "defensive" in the sense that losses occur when consumption growth is high and M is low while (2) is "aggressive" in the sense that losses occur when M is high and consumption growth low. Then it follows that

$$P_2 < \frac{1-E[Loss_2]}{1+r} = \frac{1-E[Loss_1]}{1+r} < P_1$$

But as $E[Loss] = Pr_{Default} * LGD$ and setting Loss Given Default constant with expected losses equal, this means that they have the same objective default probability but Prices are different and so yields must be different.